

Find the Maclaurin Polynomial of the stated degree with the Lagrange form of the remainder:

(1) $f(x) = \frac{1}{x-2}$; degree 4

(2) $f(x) = e^{-x}$; degree 5

(3) $f(x) = \cos x$; degree 6

(4) $f(x) = \sin hx$; degree 4

(5) $f(x) = (1+x)^{3/2}$; degree 3

Find the Taylor Polynomial of the stated degree at the given number a with Lagrange form as remainder:

(6) $f(x) = x^{3/2}$; $a = 4$; degree 3

(7) $f(x) = \sin x$; $a = \frac{1}{6}\pi$; degree 3

(8) $f(x) = \ln x$; $a = 1$; degree 5

(9) $f(x) = \ln \cos x$; $a = \frac{1}{3}\pi$; degree 3

(10) Apply Taylor's formula to express the polynomial $P(x) = x^4 - x^3 + 2x^2 - 3x + 1$ as a polynomial in powers of $(x-1)$.

Answer Key

$$(1) P_4(x) = -\frac{1}{2} - \frac{1}{4}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 - \frac{1}{32}x^4; R_4(x) = \frac{x^5}{(2-2)^6}; 2 \text{ between } 0 \text{ and } x$$

$$(2) P_5(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!}; R_5(x) = \frac{e^{-2}}{6!}x^6 \quad 0 < 2 < x$$

$$(3) P_6(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}; R_6(x) = \frac{\sin z}{7!}x^7 \quad 0 < z < x$$

$$(4) P_4(x) = x + \frac{1}{6}x^3; R_4(x) = \frac{1}{120}(\cos n z)x^5 \quad 0 < z < x$$

$$(5) P_3(x) = 1 + \frac{3}{2}x + \frac{3}{8}x^2 - \frac{1}{16}x^3; R_3(x) = \frac{3}{128}(1+z)^{-5/2}x^4 \quad 0 < z < x$$

$$(6) P_3(x) = 8 + 3(x-4) + \frac{3}{16}(x-4)^2 - \frac{1}{128}(x-4)^3; R_3(x) = \frac{3(x-4)^4}{128z^{5/2}} \quad 4 < z < x$$

$$(7) P_3(x) = \frac{1}{2} + \frac{1}{2}\sqrt{3}\left(x - \frac{1}{6}\pi\right) - \frac{1}{4}\left(x - \frac{1}{6}\pi\right)^2 - \frac{1}{12}(\sqrt{3})\left(x - \frac{1}{6}\pi\right)^3;$$

$$R_3(x) = \frac{1}{24}\sin z\left(x - \frac{1}{6}\pi\right)^4 \quad \frac{\pi}{6} < z < x$$

$$(8) P_5(x) = x - 1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \frac{1}{5}(x-1)^5; R_3(x) = -\frac{1}{6}z(x-1)^6 \quad 1 < z < x$$

$$(9) P_3(x) = -\ln 2 - \sqrt{3}\left(x - \frac{1}{3}\pi\right) - 2\left(x - \frac{1}{3}\pi\right)^2 - \frac{4}{3}\sqrt{3}\left(x - \frac{1}{3}\pi\right)^3$$

$$R_3(x) = -\frac{1}{12}(3 \sec^4 z - 2 \sec^2 z)\left(x - \frac{1}{3}\pi\right)^4 \quad \frac{1}{3}\pi < z < x$$

$$(10) 2(x-1) + 5(x-1)^2 + 3(x-1)^3 + (x-1)^4$$